

# CBCS SCHEME

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18AI56

## Fifth Semester B.E. Degree Examination, Jan./Feb. 2023 Mathematics for Machine Learning

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

### Module-1

- 1 a. Find the general and particular solution of the system of equations :
- $$\begin{aligned} -2x_1 + 4x_2 - 2x_3 - x_4 + 4x_5 &= -3 \\ 4x_1 - 8x_2 + 8x_3 - 3x_4 + x_5 &= 2 \\ x_1 - 2x_2 + x_3 - x_4 + x_5 &= 0 \\ x_1 - 2x_2 - 3x_4 + 4x_5 &= a. \end{aligned}$$
- (10 Marks)
- b. Define a basis, rank and inner product space with an example. (06 Marks)
- c. Determine a basis for a vector subspace  $U \subseteq \mathbb{R}^5$ , spanned by the vectors  $\{x_1, x_2, x_3, x_4\} \in \mathbb{R}^5$ , where  $x_1 = [1, 2, -1, -1, -1]^T$ ,  $x_2 = [2, -1, 1, 2, -2]^T$ ,  $x_3 = [3, -4, 3, 5, -3]^T$ ,  $x_4 = [-1, 8, -5, -6, 1]^T$ . (04 Marks)

OR

- 2 a. If  $u = [2, -5, -1]^T$  and  $v = [-7, -4, 6]^T$  then :
- i)  $u \cdot v$     ii)  $u + v$     iii)  $\|u\|$     iv)  $\|v\|$     v)  $\|u + v\|$ . (10 Marks)
- b. For  $x, y \in v$  where  $v$  is an inner product space then define distance between  $x$  and  $y$ . Also find the distance between  $(7, 1)$  and  $(3, 2)$ . (05 Marks)
- c. Define angle between two vectors from Cauchy – Schwartz inequality. Hence compute angle between  $[1, 1]^T$  and  $[1, 2]^T$ . (05 Marks)

### Module-2

- 3 a. For the matrix  $A = \begin{bmatrix} 2 & 6 & 1 \\ 0 & 1 & 4 \\ -8 & 0 & -1 \end{bmatrix}$ . Find  $|A|$  and trace  $(A)$ . (05 Marks)
- b. Find the Eigen values, Eigen vectors, and Eigen spaces for the matrix  $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ . (06 Marks)
- c. Find the singular value decomposition of  $A = \begin{bmatrix} 0 & 1 & 0 \\ -2 & 1 & 0 \end{bmatrix}$ . (09 Marks)

OR

- 4 a. Explain the terms orthogonal and orthonormal vectors. (04 Marks)
- b. If  $a = [-2, 1]^T$ ,  $b = [-3, 1]^T$ ,  $c = [4/3, -1, 2/3]^T$  and  $d = [5, 6, -1]^T$ , then compute.
- i)  $\left(\frac{a \cdot b}{a \cdot a}\right) \cdot a$
- ii) Find a unit vector 'u' in the direction c
- iii) Show that 'd' is orthogonal to 'c'. (06 Marks)
- c. Compute the singular value decomposition of a matrix  $A = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$ . (10 Marks)

**Module-3**

- 5 a. Obtain the Maclaurin's series of  $f(x) = \sqrt{1 + \sin 2x}$ . Draw the graphs of  $f(x) = f(0)$ ,  
 $f(x) = f(0) + \frac{x}{1!} f'(0)$  and  $f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0)$ . (07 Marks)
- b. Define gradient of the vector valued functions  $f(x_1, x_2)$  and hence find gradient of  
 $f(x_1, x_2) = x_1^2 x_2 + x_1 x_2^3$ . (07 Marks)
- c. Compute the derivative of the function  $h(x) = g[f(x)]$  where  $g[f(x)] = [f(x)]^4$  and  
 $f(x) = 2x + 1$ . (06 Marks)

**OR**

- 6 a. If  $f(x, y) = x^2 + 2y$ ; where  $x_1 = \sin t$  and  $x_2 = \cos t$ , then find  $\frac{df}{dt}$ . (06 Marks)
- b. Compute the gradient of  $h$  with respect to  $t$ , for the function  $h : \mathbb{R} \rightarrow \mathbb{R}$ ,  $h(t) = (f \circ g)(t)$  with  
 $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ ,  $g : \mathbb{R} \rightarrow \mathbb{R}^2$  defined by  $f(x) = e^{xy^2}$   $x = \begin{bmatrix} x \\ y \end{bmatrix} = g(t) = \begin{bmatrix} t \cos t \\ t \sin t \end{bmatrix}$ . (06 Marks)
- c. For the linear regression model  $f(x) = \sqrt{x^2 + e^{x^2}} + \cos(x^2 + e^{x^2})$ . Compute the gradient and  
 $\frac{\partial f}{\partial x}$  by working backward. (08 Marks)

**Module-4**

- 7 a. Let  $x$  and  $y$  are two independent variables. If variance of  $(2x - y) = 6$  and variance of  
 $(x + 2y) = 9$ . Find the variance of  $x$  and variance of  $y$ . (06 Marks)
- b. A joint probability distribution is given by the following table.

X \ Y	-3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

- Find variance of  $x$  and  $y$  and write the marginal distributions  $p(x)$  and  $p(y)$ . (07 Marks)
- c. State the following :
- Sum rule for discrete and continuous random variables
  - Product rule which relates the joint distribution to the conditional distribution.
  - Baye's theorem
  - Likelihood  $P\left(\frac{y}{x}\right)$
  - Posterior  $P\left(\frac{x}{y}\right)$ . (07 Marks)

**OR**

- 8 a. A bag contain 2 mango and 3 apples and a bag B contains 4 mango and 5 apples. One fruit is  
drawn at random from one of the bags and it is found to be apple. Find the probability that  
the apple fruit is drawn from bag B. (06 Marks)
- b. Given a real valued function  $f(x) = a g(x) + b h(x)$  where  $a, b \in \mathbb{R}$  and  $x \in \mathbb{R}^D$ , then prove that  
 $E_x[f(x)] = a E_x[g(x)] + b E_x[h(x)]$ . (04 Marks)
- c. For any two random variable, prove that
- $E(X + Y) = E(X) + E(Y)$
  - $E(X - Y) = E(X) - E(Y)$
  - $V(X + Y) = V(X) + V(Y) + \text{COV}(X, Y) + \text{COV}(Y, X)$
  - $V(X - Y) = V(X) + V(Y) - \text{COV}(X, Y) - \text{COV}(Y, X)$ . (10 Marks)

**Module-5**

- 9 a. Distress for the function  $f(x) = x^4 + 7x^3 + 5x^2 - 17x + 3$ , the convexity and concavity in the interval  $(-6, 2)$ . Further graphically represent the negative gradients and global minimum. (10 Marks)

- b. For a quadratic function in two dimensions  $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \frac{1}{2} \begin{bmatrix} x \\ y \end{bmatrix}^T \begin{bmatrix} 2 & 1 \\ 1 & 20 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} 5 \\ 3 \end{bmatrix}^T \begin{bmatrix} x \\ y \end{bmatrix}$  with gradient  $\begin{bmatrix} x \\ y \end{bmatrix}^T \begin{bmatrix} 2 & 1 \\ 1 & 20 \end{bmatrix} - \begin{bmatrix} 5 \\ 3 \end{bmatrix}^T$  initial location,  $x_0 = [-3, -1]^T$  and  $r = 0.085$ . Apply gradient descent algorithm which converges to minimal value perform two iterations. (10 Marks)

**OR**

- 10 a. Define convex optimization problem, further discuss the convexity of  $x \log_2 x$  between the points  $x = 2$  and  $x = 4$ . Also plot the function and tangent at  $x = 2$ . (10 Marks)
- b. Write a brief note on concavity and convexity with suitable examples. Further discuss the convexity of  $y = 3x^2 - 5x + 2$  with graph. (10 Marks)

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